Guidance of light along an air-column in a new class of optical fibers

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Abstract

Photonic crystals are described. A numerical method for calculating the band-structures for such crystals is explained, and the appearance of forbidden frequency gaps is shown. This is used to explain the principle of a completely new kind of optical fibers with novel qualities. It is found that these fibers may guide light along a low index air-column, in accordance with the fact that they do not guide by total internal reflection.

Introduction

The forbidden electron energy gaps in semiconductor crystals have led to the tremendous success of semiconductor devices. In 1987 Yablonovitch suggested that, by analogy, periodic dielectric structures could exhibit photonic bandgaps¹ (PBGs), that is frequency intervals where no extended electromagnetic solutions exist. One of the appealing aspects of this is that by embedding an excited atom in such a properly designed periodic material (a photonic crystal) the relaxation rates of the atom may significantly be altered. The requirement for this is a spectral overlap between the relaxational transitions of the atom and the PBG of the photonic crystal. Hereby, the number of states available for the excited atom to couple to may be dramatically reduced.² As well as for single atoms, photonic crystals may be used for spontaneous emission control in semiconductors. This potential of emission control has led to a large research interest in photonic crystals for use in low-threshold lasers.³⁻⁵

Another very exciting aspect of photonic bandgap materials is their potential of providing new means of electromagnetic waveguiding. An electromagnetic field reaching the surface of a photonic crystal, with a frequency inside a PBG of the crystal will be reflected back by Bragg-diffraction. However, by locally breaking the periodicity of the photonic crystal (creating a socalled defect), localized field solutions (with frequencies falling inside the bandgap of the bulk photonic crystal) may exist in this spatial defect. It has recently been demonstrated that by this principle, it is possible to guide electromagnetic waves around 90° sharp bends with 100% transmission in a photonic crystal configuration, where the periodicity is restricted to two dimensions.^{6,7} Although not demonstrated at optical wavelengths, but only at millimeter waves, such planar waveguides appear very interesting for future large-scale integrated optics. The problems related to the realization of photonic crystals operating at optical wavelengths is the requirements of large index contrasts between the constituting materials as well as a very precise material morphology with a periodicity on the scale of the optical wavelength. These requirements have proven very difficult to achieve for planar photonic crystal structures. However, in another type of PBG waveguide also having a two-dimensional periodicity, light may be guided along the invariant direction of the crystal. This type of design is favorable for long distance waveguiding, and known as photonic crystal fibers.^{8,9} Very recently such waveguides have been used for the first experimental demonstration of waveguiding by the photonic bandgap effect at optical wavelenghts.¹⁰ We believe that these new waveguides have a large potential in both telecommunication and sensor areas, and we will be addressing the underlying physical principles of their operation in this article.

Theory

In 1990 the first efficient method for calculating photonic bandgap structures of photonic crystals was described.¹¹ This method basically assumes the crystal to extend indefinitely in space. Then any solution is extended, and can be described as a sum of plane waves, due to the crystal periodicity.

A two dimensional photonic crystal is a dielectric structure which is periodic in two dimensions, and invariant in the third dimension, see Fig. 1.



Fig. 1. The geometry of a 2D photonic crystal. The periodicity of the structure is described through the two simple lattice vectors \mathbf{R}_1 and \mathbf{R}_2 . In this case they define a triangular lattice structure. The invariant direction is normal to the plane defined by the lattice vectors.

The lattice defining the periodicity of the structure is defined by its lattice vectors $\mathbf{R}=n_1\mathbf{R}_1+n_2\mathbf{R}_2$, where n_1 and n_2 are integers. Then extended field solutions inside the crystal may be described as a plane wave multiplied by a Bloch function with the same discrete translational symmetry as the lattice structure.¹² Therefore, any extended field solution (exemplified by the magnetic field) can be written as the infinite sum:

$$\mathbf{H}(\mathbf{k},\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{\mathbf{G}} \sum_{j=1}^{2} h_j(\mathbf{k}+\mathbf{G}) \mathbf{e}_j(\mathbf{k}+\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}}$$
(1)

Here **G** defines the reciprocal lattice, $\mathbf{G} \cdot \mathbf{R} = N 2\pi$, where *N* is an integer. The vector $(\mathbf{k}+\mathbf{G})$ and the unit vectors $\mathbf{e}_1(\mathbf{k}+\mathbf{G})$ and $\mathbf{e}_2(\mathbf{k}+\mathbf{G})$ define a triad.¹³ **k** is the wavevector of the plane wave. The actual field (and eigenfrequencies), is found by expressing Maxwell's equations in the form of a Hermitian operator equation:

$$\nabla \times \left[\frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r})\right] = \left(\frac{\omega}{c}\right)^2 \mathbf{H}(\mathbf{r})$$
(2)

To solve this equation we Fourier-transform the inverse dielectric function $1/\epsilon(\mathbf{r})$ as well:

$$\frac{1}{\varepsilon(\mathbf{r})} = \sum_{\mathbf{G}} \varepsilon_{\mathbf{G}}^{-1} e^{i\mathbf{G}\cdot\mathbf{r}}$$

$$\varepsilon_{\mathbf{G}}^{-1} = \frac{1}{a_{c}} \int_{a_{c}} d^{2}\mathbf{r} e^{-i\mathbf{G}\cdot\mathbf{r}} \frac{1}{\varepsilon(\mathbf{r})}$$
(3)

Here a_c is indicated in Fig. 1 and the sum is in principle over all possible **G** vectors. One finds the dielectric constants, by either using an FFT, or by finding analytical expressions for the Fourier coefficients. We have used the latter technique, finding the analytical expressions by an enhancement of the method given in Ref. [14].

For a given choice of the reciprocal lattice vector \mathbf{G} we then find the following dependency from (1) and (2):

$$\sum_{\mathbf{G}} \begin{cases} \mathbf{e}_2 \cdot \mathbf{e}_2' & -\mathbf{e}_2 \cdot \mathbf{e}_1' \\ -\mathbf{e}_1 \cdot \mathbf{e}_2' & \mathbf{e}_1 \cdot \mathbf{e}_1' \end{cases} \cdot \begin{cases} h_1' \\ h_2' \end{cases} \varepsilon_{\mathbf{G}-\mathbf{G}'}^{-1} L = \left(\frac{\omega}{c}\right)^2 \begin{cases} h_1 \\ h_2 \end{cases} \quad (4)$$

where $L=|\mathbf{k}+\mathbf{G}| |\mathbf{k}+\mathbf{G}'|$. Inside the matrices the operand is the vector $(\mathbf{k}+\mathbf{G}')$ if a prime (') is given, and the vector $(\mathbf{k}+\mathbf{G})$ if no prime is given. We write N such equations (one for each of the N shortest **G** vectors), truncating the sum to the same N **G** vectors, $\mathbf{G}_i=\mathbf{G}'$, to obtain a simple Hermitian eigenvalue matrix equation using a basis of N plane waves.

The first Brillouin zone is the set of points in the reciprocal lattice plane, which are closer to the lattice grid point **G=0**, than to any other reciprocal lattice grid points (see Fig. 2). The first Brillouin zone can be constructed from the irreducible Brillouin zone (shown with dark shading in Fig. 2) through the use of mirroring and rotational symmetry.¹² For finding the possible PBGs of a photonic crystal, one solves the eigenvalue-problem for all the different values of \mathbf{k}_{π} (\mathbf{k}_{π} is \mathbf{k} projected on the plane defined by the reciprocal lattice vectors) along the boundary of the irreduc-



Fig. 2. Construction of the first Brillouin zone shown with light shading. The dark spots represent the reciprocal lattice points. The Brillouin zone is the set closest to **0**. The irreducible Brillouin zone is shown with dark shading. In this case the periodicity is described by a triangular lattice.

ible Brillouin zone^{12,15} (see Fig. 2). Any frequency intervals, where no solutions are found define the PBGs of the photonic crystal.

For propagation restricted to the plane defined by the lattice vectors **R** (that is $\mathbf{k}=\mathbf{k}_{\pi}$) the two different plane wave directions in (4) decouple into TM solutions (*j*=1 corresponding to the $\mathbf{e}_2 \cdot \mathbf{e}_2$) and TE solutions (*j*=2 corresponding to the term $\mathbf{e}_1 \cdot \mathbf{e}_1$). Here we define TM solutions as solutions having the electric field along the invariant direction, while TE solutions have the electric field in the plane defined by the lattice vectors. For $\mathbf{k}\neq\mathbf{k}_{\pi}$ the equations no longer decouple, and the complete set of equations in (4) must be solved.

We conclude that for a given wave-vector we need to solve an $N \times N$ eigenvalue problem for in-plane propagation, while a full three dimensional field analysis demands the solution of a $2N \times 2N$ eigenvalue problem. In any case the computational demands increases quickly with the number of plane waves used. However, one may obtain better convergence for a given N, by creating the dielectric matrix $\overline{\overline{\epsilon}}_{G-G'}$, inverting this matrix to give $\overline{\overline{\eta}}_{G-G'}$, and then replacing $\overline{\overline{\epsilon}}_{G-G'}^{-1}$ with $\overline{\overline{\eta}}_{G-G'}$.^{16,17} This is commonly called the Hammatha dill concurrent to give the term propagation.

Ho method.¹¹ For our purpose this technique greatly reduces the computational time, since we only need one matrix inversion for each structure, while a large number of matrix equations need to be solved.

Basic photonic crystal results

When propagation is restricted to the plane defined by the lattice vectors (TE- or TM-case) it has been found that a quite large refractive index contrast must exist for PBGs to appear. To obtain a complete PBG (a PBG for arbitrary in-plane polarization), it has been found that one needs a refractive index ratio of at least 2.66.¹⁸ An example of a photonic crystal exhibiting a PBG, is a GaAs substrate (refractive index 3.6), with a triangular arrangement of circular air-columns. The band diagram for such a



Fig. 3. Triangular lattice structure with circular air holes on a GaAs background with the refractive index 3.6. The air holes have diameter 0.48 A, where A is the lattice constant. $\mathbf{k}=\mathbf{k}_{\pi}$ (in plane propagation), and the bands are split up into TE and TM modes. A complete photonic bandgap is found between ω_{\min} and ω_{\max} corresponding to $\Lambda/\lambda = 0.43 - 0.52$.

structure is shown in Fig. 3. The first axis shows \mathbf{k}_{π} along the boundary of the irreducible Brillouin zone. Notice the symmetry points, 0, M, and X, defined in Fig. 2. The numbers on the second axis show the normalized frequency, Λ/λ , where Λ is the distance between the centers of neighboring air-columns and λ is the free space wavelength. It is noted that there is both a TE and a TM PBG. These overlap to give a complete PBG from $\lambda = \Lambda/0.43$ to $\lambda = \Lambda/0.52$.

For $\mathbf{k} \neq \mathbf{k}_{\pi}$ (we shall call this out-of-plane propagation), we may still find PBGs.¹⁹ We denote the wavevector component in the invariant direction of the photonic crystal, the propagation constant β . This definition is consistent with the propagation constant of standard optical fibers, since the fiber is in principle infinitely long in this direction. We, therefore, find the PBGs for a



Fig. 4. Band-structure in a triangular lattice structure consisting of air rods on a silica background (refractive index=1.45). The filling fraction of air is 65 %. $\beta = (4\pi)/\Lambda$. Two photonic bandgaps are indicated. Notice that a minimum frequency limit exists below which no extended solutions exist for $\beta \neq 0$.

given value of β , by varying \mathbf{k}_{π} along the boundary of the irreducible Brillouin zone as before.

An interesting aspect of out-of-plane propagation is that it has been reported that PBGs can be found for much smaller refractive index ratios than for in-plane propagation. Thus PBGs have been reported for circular air-columns in silica (with a refractive index of 1.45).⁸ In Fig. 4 two such PBGs are shown. Notice that when $\beta \neq 0$ there is a minimum frequency below which no extended solutions exist. This is similar to the radiation line of a homogeneous material, and corresponds to the effective refractive index of the cladding in a standard optical fiber.

Photonic crystal fibers

The concept of a photonic crystal fiber (PCF) was originally proposed by Russell and co-workers, based on the finding of complete PBGs in silica with circular air-columns.⁸ The structure found to exhibit PBGs was a triangular structure as the one underlying Fig. 4 except that the air-filling fraction was only 45%. In Ref. 8 it was, therefore, suggested to make a fiber consisting of a silica/air cladding with a defect added which should break the periodicity. Such a defect could be the exclusion of the central air-column in a triangular structure.

It was quickly realized that it was difficult to produce fibers with sufficiently large air-columns for PBGs to appear.^{20,21} However, the fibers produced still guided light along the defect-core, which was explained by the core having a higher effective refractive index, than the surrounding cladding. These fibers, therefore, guide by the well-known effect of total internal reflection (TIR). Albeit this, they do exhibit many unusual properties compared with traditional optical fibers, e.g. they can be made singlemode at all frequencies.²⁰



Fig. 5. PCF design with a honeycomb cladding structure. The low index core is seen at the center of the fiber.

For the realization of truly PBG guiding fibers, we have recently presented a new PCF design^{22,23} and this has proven successful for the experimental demonstration of PBG waveguiding.¹⁰ Instead of using a triangular structure, we suggested using a honeycomb structure with an extra air-column added at the center to

act as a core-defect (see Fig. 5). We found this cladding structure to exhibit PBGs for far smaller air-columns than the triangular PCF design. Further beneficial, the air defect-core has a lower effective refractive index than the surrounding cladding. Guidance by TIR is, therefore, not possible using this design. However, we have found the honeycomb structure to exhibit PBGs for realistic air-column sizes. The honeycomb fiber design thus satisfies the basic requirements for realizing a PBG waveguide. Even though the honeycomb cladding structure is reminiscent of the triangular cladding structures in Ref. 20 and 21 the guiding principle is, therefore, fundamentally different.

Instead of showing band diagrams for a number of different β values, we choose to depict the fiber index β/κ as a function of the normalized wavelength λ/Λ , where Λ is the distance between neighboring air-column centers. Such a depiction is shown for the case where the diameter of the cladding air-columns is 0.55 Λ , and the core-column has a diameter of 0.33 Λ .

The radiation line at the top is the lowest frequency solution for a given value of β . Above this line is a semi-infinite 'bandgap', where no extended field solutions exist. In a standard optical fiber this line corresponds to the refractive index of the cladding as a function of frequency. From this it is understood that the guided TIR modes of PCFs with a high index core, are squeezed in between the radiation line, and the refractive index of silica.

Below the radiation line is depicted two PBGs (with their boundaries shown in dotted lines). These are complete PBGs with no analogy to standard optical fibers. Inside the PBGs are shown the localized modes. These modes are not extended solutions of the cladding structure - since no extended solutions exist inside the PBGs. Instead they are solutions localized to the vicinity of the core-defect.

From Fig. 6 it is seen that the fiber has a guided mode which traverses the 'first' bandgap from $\lambda/\Lambda \approx 0.3$. Guided modes are also seen to 'start' inside the 'second' bandgap at $\lambda/\Lambda \approx 0.35$. These are actually several different modes, which lie close in frequency. These modes extend to quite high frequencies $\lambda/\Lambda \approx 0.1$.

Here we are particularly interested in the degenerate mode in the first bandgap, since the fiber is monomode here. The large down-



Fig. 6. Modal analysis of PBG-guiding honeycomb PCF. The cladding air-columns have diameter 0.55 Λ , while the central core-defect has diameter 0.33 Λ . Notice the localized modes inside the PBGs. The propagation constant is indicated by the $\beta \Lambda = 5$, 10, 15,...curves.



Fig. 7. Field analysis for the guided mode in the first bandgap in Fig. 6. The solution is for the free space wavelength $\lambda \approx \Lambda$, where the guided mode is approximately in the middle of the bandgap, see Fig. 6. Here **D**·**E*** is plotted for the sum of the two degenerate core-modes. Notice that the field is localized in the low-index core-region.

ward bending of the core-mode at large wavelengths suggests that this type of fiber may make it possible to have fibers with large dispersion²⁴ ($D=-\lambda/c$ ($d^2n/d\lambda^2$), which is truly a remarkable feature. At shorter wavelengths there appear to be a small upward bending suggesting that also near-zero dispersion below zero is possible. It, therefore, appears that completely new dispersion management systems are possible using these fibers.

Also notice that the index difference between the guided mode and the nearest cladding mode (The PBG-boundary) is quite large when $\lambda/\Lambda \approx 1$. This led us to expect good bending properties for these fibers. Furthermore, the all-silica principle of these optical fibers should give low scattering losses. Finally, it is noticed that the mode-index is well below 1.45. This suggest that new sensor systems may be possible using these fibers.

To verify that the field is localized in the low-index core region, we show the calculated field for $\lambda/\Lambda \approx 1$ in Fig. 7. Guiding light by the PBG effect, it becomes possible to guide light in low index regions.⁶

To summarize, it can be said that PBG guiding optical fibers form a new class of optical fibers. It should, however, be apparent that this completely new guiding principle offers novel possibilities in optical fiber technology.^{9,25,24} Only the future can tell exactly how important these possibilities will become for the fabrication of optical fibers.

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