Classical interference is a quantum effect

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Abstract

Motivated by the lack of a natural microscopic mechanism for production of optical fields with non-vanishing field amplitudes, this paper investigates the claim that such fields do not exist, i.e., that, strictly speaking, classical optics and most laser theory is wrong. We investigate interference between independent light fields, and we show that due to the so-called back-action in quantum measurements, such fields become quantum entangled when they are detected, and quantum effects conspire to produce measurements results indistinguishable from the ones predicted by classical optics. Classical optics is of course an excellent theory, but the reason for that is 'more quantum' than one would have expected.

Introduction

Quantum theory is the correct microscopic theory of light and matter, but it is not always possible to carry out a complete quantum analysis of a given problem, and it is not always necessary - classical treatments are often sufficient. If not at every application, then at least at certain key developments, a classical treatment should be justified by arguments for why it produces the same results as the quantum theory.

Section II provides a brief introduction to the quantum theory of light and matter. Reasons for classical theory being sometimes adequate are presented, and it is claimed that these reasons do not hold for the classical theory of light. Section III presents a quantum mechanical analysis of the photodetection of light from two independent light sources. Due to the subtle role of measurements in quantum theory, interference appears just like in the classical theory, even though neither of the fields have an amplitude or a phase. Section IV presents a brief formal proof of the wide validity of classical optics.

Quantum theory of light and matter

The quantum theory was presented in the 1920'es in a form that differs only little from its present formulation. Our knowledge about a physical system is represented by a wave function or a state vector, which obeys a certain equation of evolution, the Schrödinger Equation. Physical observables are taken over from the classical theory, but their mathematical properties are slightly changed, for example the product of position and momentum of a particle depends on the order of the factors, and we represent physical observables as operators, acting on the wave function to produce the predictions of the theory. The values observed in an experiment are predicted by the theory in terms of probability distributions, and the meeting point for quantum and classical theory is the comparison of mean values predicted by quantum theory with the values obtained from classical theory. A classical theory is adequate if it predicts the quantum mean value, or expectation value, with good precision. Atoms have discrete energy values for their various excited electronic states, as postulated already in Bohr's model for the atom in 1913, and light is also described quantum mechanically by a state vector and by observables like the total energy which has discrete values separated by the energy of a single photon, $\hbar\omega$, where $\hbar \sim 10^{-34}\ J{\cdot}s$ is Planck's constant, and where ω is the optical frequency. The operator character of observables in optics have various consequences, for example in terms of the precision by which one may measure field amplitudes and intensities, see for example Ref. 1.

Light is emitted by atoms, when the electrons rearrange themselves in a state with a lower energy, and the photon energy equals the change of atomic energy in the process. Although we talk about dipole radiation and we write down the atom-light interaction as a coupling of the polarization of the atom with the electric field amplitude of the light, $-\mathbf{D}\cdot\mathbf{E}$, the microscopic process differs substantially from the emission of radiation from, e.g., a classical antenna. Neither the initial (excited) state of the atom $|e\rangle$ nor the final (ground) state $|g\rangle$ has a dipole moment, and neither the zero-photon state $|0\rangle$ nor the one-photon state $|1\rangle$ has a mean electric field amplitude.

In the course of time a superposition state

$$c_0|e\rangle|0\rangle + c_0|g\rangle|1\rangle \tag{1}$$

evolves continuously between the product states $|e\rangle|0\rangle$ and $|g\rangle|1\rangle$, and at no point, neither before, after or during the process, is there a non-vanishing mean value of the atomic dipole or of the electric field amplitude. The state (1) is an entangled state, where the expectation value of either quantity vanishes, because the other system occupies orthogonal states.

It is quite easy to see, that for any system emitting light in the optical domain, the creation of a photon is accompanied by a change of state of the emitter, and if the 'whole laboratory' is described quantum mechanically, there will never appear such a thing as a mean field amplitude.[2-4] We are not referring to the fact that the amplitude averages to zero over an optical period, or the possibility that uncertainty about the phase, e.g., due to phase fluctuations, renders the phase unknown, and hence also the slowly varying envelope amplitude zero on average - the claim is that instantaneously, there is no possibility to even imagine a non-vanishing field amplitude. There will be plenty of light, the intensity is represented by the population of states with non-vanishing photon number ($|c_1|^2$ in (1)), and there will be plenty of entanglement of the different systems, so that quantities like products of two different field amplitudes (E_1E_2) do not vanish in general.

If there are no mean fields, there is no mean polarization induced in media illuminated by light, and the classical theory of light propagation and refraction is not merely a theory of “mean values of the quantum theory”. However, the spatial and temporal light propagation is the same for quantum mechanical operators and for classical quantities, i.e., the alleged mean values of field and dipole operators. In the linear regime the steady state atomic dipole operator is proportional to the field operator of the driving field with the same constant of proportionality as between the mean dipole and an injected classical field, and the same observable refraction is predicted. And, as Dirac once formulated it, “interference is every single photon interfering with itself”, a single photon has no amplitude, but in an interferometer different paths lead to interfering (quantum) amplitudes, and the usual interference pattern and, e.g., frequency resolution, appears.

Is classical interference the consequence of single-photon quantum interference, or is the superposition principle for single photons inherited from Maxwell’s equations, and thus classical interference comes first? This is a *hen-and-the-egg* question, and we shall not insist here that self-interference effects, where the light comes from a single source, reflect any particular quantum nature of light.

Let us instead turn our attention to light from two different sources. Consider two frequency-stabilized lasers, with their laser beams directed onto a single detector which records the incident intensity as a function of time. In the classical description the two laser beams have well-defined amplitudes and phases, and if they have slightly different frequencies, the fields interfere to produce a beat-node in the recorded intensity. Most opticians are aware of such interference experiments (somebody they know - in Danish: “deres gamle moster” - saw the experiment, herself); the interference is observed, but what is its origin? The product of the field operators for two different lasers which are not locked to one another (the gain media are not mutually correlated or entangled), is a quantum mechanical operator with vanishing mean value according to the above argument, and we do not predict interference in such an experiment. Is there a mysterious spontaneous symmetry breaking mechanism in lasers that makes the light coherent when it is intense enough? The next section will show that such a mechanism is not necessary at all. Lasers without mean amplitudes interfere perfectly, even though they have not been locked to each other. The interference comes by itself when we look for it!

Photodetection

Systems which decay by emission of radiation are traditionally described by rate equations or by master equations and density matrices, which average over the uncertainty in time of the emission events. About a decade ago, we introduced an alternative method, which operates with a real time simulation of the detection record, in which detection events are simulated, and in which the quantum state of the system, e.g., an atom, is updated according to the measurement.[5,6] As exemplified below, when it is randomly decided that a photon has been detected, the state vector of the system ’jumps’ to the component corresponding to that event, and between jumps, the absence of photodetector clicks causes an evolution of the state vector (a null-measurement is also a measurement), for details, see Refs. 5, 6. This Monte Carlo wavefunction method has many applications: 1) the propagation of a wave function, conditioned on measurements, is much easier than the propagation of a density matrix if the system has many states,[7] 2) it leads to an interpretation of the noise in photodetection records,[6] 3) it provides an understanding of the dynamics of dissipative quantum systems, e.g. towards dark states,[5] lasing without inversion,[8] etc.

Detection on a single mode field

The number of photons in a single mode in a cavity are represented by the operator a^+a, with mean value $\langle a^+a\rangle$, a is the annihilation operator and a^+ is the creation operator for photons in the cavity mode. If it is a lossy cavity, the field leaks out, and we obtain an exponential decrease of the in-cavity intensity. Instead of this average decay curve, we may imagine an idealized photodetector outside the cavity which registers the photons leaving the cavity, and which for each detector “click” causes the application of the annihilation operator a on the state of the field in the cavity. These quantum jumps occur with a rate $\Gamma\langle a^+a\rangle$, and between jumps the state vector evolves according to a Schrödinger equation with a non-hermitian hamiltonian, $H_{eff} = (\omega - ½ i\Gamma)a^+a$, and the state vector has to be renormalized whenever needed for the computation of expectation values, e.g., jump rates.

It is illustrative to apply this scheme to a single mode field with two different initial states, a) a number state $|n\rangle$ and b) a coherent state .

The two elements of the dynamics have the following effect on the number state:

$$\begin{aligned} \text{jump: } |n\rangle &\rightarrow |n-1\rangle \\ \text{no jump: } |n\rangle &\rightarrow |n\rangle \end{aligned} \tag{2}$$

where an unimportant phase-factor has been omitted.

The coherent state is an eigenstate of the annihilation operator, but the anti-hermitian part of H_{eff} causes the different number state components of $|\alpha\rangle$ to decay with rates proportional to n, and after a time interval of duration one finds a modified coherent state:

$$\begin{aligned} \text{jump: } |\alpha\rangle &\rightarrow |\alpha\rangle \\ \text{no jump: } |\alpha\rangle &\rightarrow \left|\alpha e^{-i(\omega - i\Gamma/2)\tau}\right\rangle \end{aligned} \tag{3}$$

In case of the number state, we must introduce a back-action (jump) on the field state at each click, and only by accumulating the effect of all jump events in the state vector, and by averaging over many realizations, we obtain the exponentially decreasing detection signal. The coherent states change continuously irrespective of jumps occurring or not: The mean photon number decays exponentially with a rate Γ, and so does the click-rate.

Two modes, entanglement

We now consider light emitted from two single mode cavities and combined into two new beams by a 50/50 loss-less beamsplitter, see Fig. 1. The relevant combinations of source annihilation operators a and b are $c = (a+b)/\sqrt{2}$ for the field impinging on one of the detectors and $d = (a-b)/\sqrt{2}$ for the field on the other one.

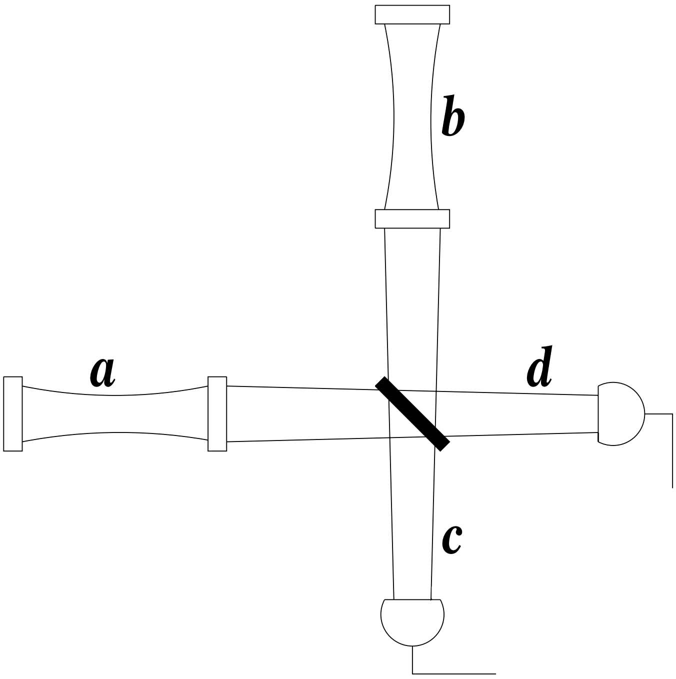

Fig. 1. Output beams from two cavities are mixed by a loss-less beam splitter and the intensities of the resulting beams are measured by two photodetectors.

Assume that the modes have different frequencies ω_a and $\omega_b = \omega_a + \Delta$, and that they are populated by n photons each at $t = 0$, $|\psi(t=0)\rangle = |n, n\rangle$. Both cavities are damped with the same decay constant Γ.

We examine the evolution by a quantum jump simulation appropriate to the detection scheme:[5] a wavefunction is propagated with the effective hamiltonian

$$H_{eff} = \hbar\omega_a\left(a^+a + \frac{1}{2}\right) + \hbar\omega_b\left(b^+b + \frac{1}{2}\right) - i\hbar\frac{\Gamma}{2}\left(a^+a + b^+b\right) \quad (4)$$

and quantum jumps $|\psi\rangle \rightarrow c|\psi\rangle$ or $|\psi\rangle \rightarrow d|\psi\rangle$ occur with $|\psi\rangle$-dependent rates $\gamma_c = \Gamma\langle\psi(t)|c^+c|\psi(t)\rangle$ and $\gamma_d = \Gamma\langle\psi(t)|d^+d|\psi(t)\rangle$. The action of either of the jumps is to reduce the total photon number by unity. Hence starting from a state with definite total photon number, $N=2n$, at a later time t the field state of the two modes is still an eigen-state of the total photon number operator a^+a+b^+b. This implies that at no time of the simulation will there be a non-vanishing mean value of any of the field amplitudes, which are proportional to annihilation and creation operators, and which thus require that the system populates states with different photon numbers, simultaneously.

We write the wavefunction as

$$|\psi(t)\rangle = \sum_{m=-N(t)}^{N(t)} c_m(t)\left|\frac{N(t)+m}{2}, \frac{N(t)-m}{2}\right\rangle \quad (5)$$

where $N(t)$ is the total number of photons at time t, and the sum over m reflects our uncertainty about the precise division of the N photons between the two cavities. As the anti-hermitian part of H_{eff} acts identically on all terms in $|\psi(t)\rangle$ it is sufficient to consider the hermitian part in the determination of the evolution of the amplitudes between jumps, and by choosing an appropriate rotating frame we obtain the equation $\dot{c}_m = im\Delta c_m / 2$ with the solution

$$c_m(t+\tau) = c_m(t)\exp\left(\frac{im\Delta\tau}{2}\right) \quad (6)$$

The total jump rate $\gamma_c + \gamma_d = \Gamma N$ is independent of the values of the amplitudes c_m. The time between jumps is therefore exponentially distributed and we select the instant of the next jump from the time increment τ solving $\exp(-\Gamma N\tau) = \varepsilon$, where ε is a random number between zero and unity. Which one of the jumps to take is determined from the ratio between the current values of the rates $\gamma_{c(d)} = \Gamma(N/2 + (-)\mathrm{Re}[Q])$, where

$$\begin{aligned} Q &= \langle\psi(t+\tau)|a^+b|\psi(t+\tau)\rangle \\ &= \sum_{m=-N}^{N-2}\sqrt{\frac{N+m}{2}+1}\sqrt{\frac{N-m}{2}}\,c_{m+2}(t)^* c_m(t)\exp(-i\Delta\tau) \end{aligned} \quad (7)$$

The value of N is decreased by unity, and an expression analogous to (5) is again valid after the jump action of the c or d annihilation operators.

The evolution is easily implemented on a computer, and in Fig. 2 we show the outcome of a single run. In the calculation we have taken $n=5000$ photons initially in each cavity, and a frequency difference $\Delta=30\Gamma$. We plot the number of jumps of each type occurring within time windows $\delta t=0.01\Gamma^{-1}$. In repeated runs, one obtains the same picture after a short entanglement period, but the oscillations are shifted in time.

The quantum analysis now accounts for the beat node, expected from the classical description and observed in experiments: each detection event causes a change (a back-action) of the quantum state of the fields. The effect is dramatic already after the first detection, where the state vector is $|\psi\rangle = (|n-1,n\rangle \pm |n,n-1\rangle)/\sqrt{2}$. The probability that the second photon is registered in the same detector as the first one is 75%! At $t=0$, the quantity Q introduced in Eqn. (7) vanishes and we do not predict any interference, but after the first detection, $Q \cong \pm N/4$, and during the first few jumps $|Q|$ gradually approaches the value $N=2$. The phase of Q at this point depends on the explicit sequence of jumps. Between jumps, Eqn. (7) reveals a simple oscillatory behavior of Q, which provides the harmonic variation in intensity at the two detectors. In Fig. 3, we show the evolution of $\mathrm{Re}[Q]$ accompanying the counts represented in Fig. 2, and displaying clearly the transition between an initial randomness and a subsequent harmonic evolution. The noise in Fig. 2 reflects the count statistics for the small number of counts in the time intervals considered in the simulation. Note that it is the back-action of this noisy signal that serves to establish the very stable entanglement reflected in the smooth evolution of Q.

If the simulation had been started with both fields in coherent states, like in (3), the jumps would have no effect on the state, which would gradually progress along a time dependent product state of coherent states, and the jump probabilities would oscillate precisely as predicted by classical theory.

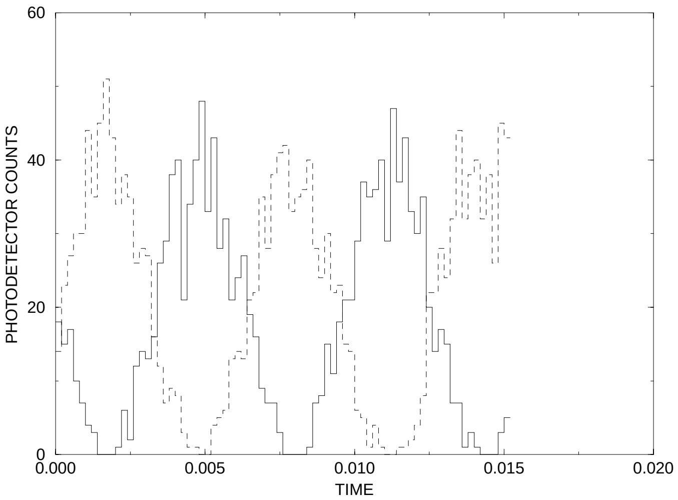

Fig. 2. Number of photons counted by the two detectors in time intervals δt=0.01 Γ^{-1}, time is given in units of Γ^{-1}, solid (dashed) line: mode-c (-d) detector. Repeated simulations with the same initial number states n=5000 in the two cavities show the same period of the two intensity signals, but the phase of the oscillations varies from simulation to simulation.

The density matrix

From number states to classical fields

In quantum descriptions of the laser a nearly Poissonian photon number distribution (diagonal density matrix) is obtained when the entanglement with the states of the gain medium has been traced out.[9] The same density matrix can be expressed in terms of coherent states $|\alpha\rangle$ with $|\alpha|^2$ equal to the mean photon number, $\bar{n}$, and the phase distributed uniformly on [0; 2π],

$$\rho = e^{-\bar{n}} \sum_n \frac{\bar{n}^n}{n!} |n\rangle\langle n| = \frac{1}{2\pi}\int_0^{2\pi} d\phi \left| \sqrt{\bar{n}} e^{i\phi} \right\rangle\left\langle \sqrt{\bar{n}} e^{i\phi} \right| \qquad (8)$$

If n is very large, we expect the same signal for the state $|n, n\rangle$ and for any state $|n_1, n_2\rangle$ where n_1, $n_2 \cong n$. It is a fundamental fact of quantum theory, the formal proof is given in the next subsection, that all probabilistic information about the outcomes of experiments is contained in the density matrix, and there is neither need nor room for assigning a particular statistical construction of ρ, such as the first or the second expression in Eqn. (8). Hence, there is complete agreement between the outcome of experiments from a statistical sampling of results based on either number states or on coherent states. For most purposes, a field in a coherent state behaves just like a classical field, hence laser experiments are perfectly described by classical fields (with á priori unknown phases) even if precise knowledge of the exact (field + surrounding matter) state vector explicitly puts the mean field to zero. For large photon numbers, the Poisson distribution is so narrow that the average over different number state components equals the result obtained with just one characteristic choice of $n \cong \bar{n}$, so even single number states yield the same interference phenomena as coherent states, as observed above.

Quantum theory of measurements

All probabilistic information about a quantum system is contained in its density matrix ρ. The expectation value of any operator A is given by $\langle A\rangle$=Tr(ρA). The probability, $P(a)$, of detecting any particular eigenvalue a of A is the expectation value of the projection operator, Λ_α, on the associated eigenspace, $P(a)=\mathrm{Tr}(\rho\Lambda_a)$ $=\mathrm{Tr}(\Lambda_a \rho\Lambda_a)$, where the last expression follows from $\Lambda_a^2 = \Lambda_a$.

Even detection records from sequences of measurements do not enable the observer to go beyond the density matrix and identify any such thing as the "true" states populated probabilistically by the system: The density matrix conditioned on the detection of a certain eigenvalue a of an operator A is given by the projection on the corresponding subspace, $\rho_c(t_1)=\Lambda_a \rho(t_1)\Lambda_a/\mathrm{Tr}(\Lambda_a \rho(t_1)\Lambda_a)$. The density matrix for an isolated system evolves according to a unitary operator, $\rho(t_1) = U(t_1,t_0)\rho(t_0)U(t_1,t_0)^\dagger$. We can therefore write the joint probability of detecting a_1 at t_1 and a_2 at t_2 as

$$P(a_2,t_2;a_1,t_1) = \mathrm{Tr}(\Lambda_{a_2} U(t_2,t_1)\Lambda_{a_1} U(t_1,t_0)\rho(t_0) \\ U(t_1,t_0)^\dagger \Lambda_{a_1} U(t_2,t_1)^\dagger \Lambda_{a_2}) \qquad (9)$$

One readily generalizes to an arbitrary detection record with eigenvalues a_n detected at the instants t_n,

$$P(a_n,t_n;a_{n-1},t_{n-1};\ldots a_1,t_1) = \\ \mathrm{Tr}(\Lambda_{a_n} U(t_n,t_{n-1})\Lambda_{a_{n-1}} \ldots \Lambda_{a_1} U(t_1,t_0)\rho(t_0) \\ U(t_1,t_0)^\dagger \Lambda_{a_1} \ldots U(t_n,t_{n-1})^\dagger \Lambda_{a_n}) \qquad (10)$$

If one considers different observables at different times, the projection operators in the expression should be replaced by the ones pertaining to the relevant operators and eigenvalues.

The entire detection record is completely classified by the value of the density matrix $\rho(t_0)$, and there is no difference between the predictions based on one or the other statistical construction of ρ!

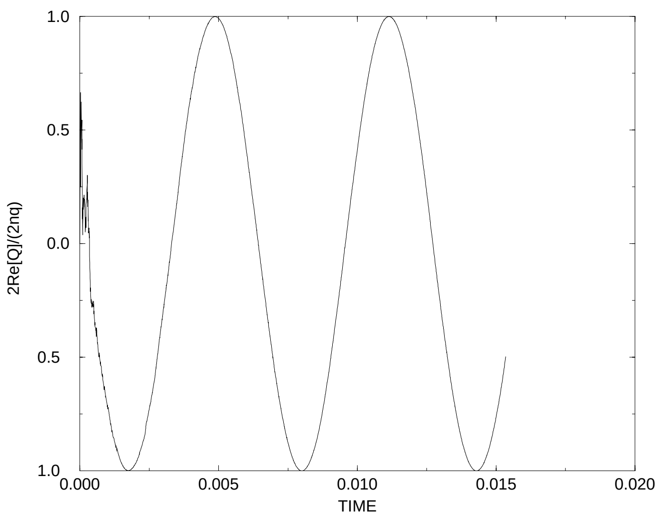

Fig. 3. Time dependence of the quantity Q, controlling the count rates in the two detected field modes. The value of 2Re[Q]/N is presented before and after every quantum jump in the simulation, but it is seen to evolve smoothly, and the noise in Fig. 2 can be ascribed to count statistics.

Discussion

From the indistinguishability of results obtained with different ensembles of wave functions with the same density matrix follows the validity of results obtained with coherent states and classical fields in many situations in optics. As illustrated in Section III, the price one has to pay for not using classical fields or coherent states is one of much more complex calculations, and one must incorporate the back-action on quantum systems from measurements. The purpose of that section, however, was not to produce quantitative results but to demonstrate that classical behavior becomes fully consistent with quantum dynamics, and the need for a postulated separation between processes that must be de-scribed by quantum theory and the ones that must be described by classical theory partly disappears.

The analysis originally grew out of the author's dissatisfaction with the text book description of lasers - it should be possible by theoretical means to come to a meaningful agreement with the classical theory without relying on authoritative statements from theorists advocating symmetry breaking, or experimentalists referring to their empirical experience. The research, and its continuation to more elaborate problems do not form a crusade against the use of classical theory. On the contrary! While adding to our understanding of some specific phenomena, these studies may also lead to new knowledge and insights in the issue of correlation, interference and entanglement.

Since this project started, related questions have appeared in atom optics and matter wave interferometry. It was, for example, a disputed questions, whether two Bose condensates that had never seen each other, would interfere. If two Bose condensed clouds of atoms emerge from different directions to overlap in space at a certain time, will the atomic density be modulated in space? If one just calculates the spatial density, assuming a fixed number of atoms in each condensate, there is absolutely no modulation; but if one simulates the detection of the atoms, the first click at any position immediately causes the remaining atoms to have a modulated density, and the modulation is enforced until a perfect sinusoidal density is found for the whole cloud.[10] The experiment[11] of course showed the interference.

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Klaus Mølmer received the DOPS Award 1998 for his work on theoretical quantum optics. He has been associate professor at Institute of Physics and Astronomy at Aarhus University since 1991 carrying out teaching and research in the areas quantum optics, laser cooling, Bose-Einstein condensates, and quantum information.